

L1-2

Point - Direction line : $\mathcal{L}(t) = \vec{a} + t\vec{v}$ $\vec{a}$ - pass through $\vec{v}$ - direction

$$\text{Coordinate form} \begin{cases} x = x_1 + at \\ y = y_1 + bt \\ z = z_1 + ct \end{cases}$$

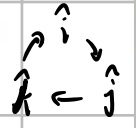
Relation between lines : ① Parallel ② Intersecting ③ Skew : if not ① or ②

Norm : $\|\vec{v}\| = \sqrt{v_1^2 + \dots + v_n^2}$ Distance : $d(P, Q) = \sqrt{\sum (y_i - x_i)^2}$ Unit vector : $\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$

Dot product : $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$ $\text{Proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \right) \cdot \vec{v}$ $\vec{u}$ on $\vec{v}$ $\vec{u} \cdot \vec{v} = \|\vec{v}\| \cdot \|\text{proj}_{\vec{v}}(\vec{u})\|$

$$\|\vec{v} - \vec{u}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\vec{u} \cdot \vec{v}$$

Cauchy - Schwarz inequality : $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \|\vec{v}\|$ $\vec{u}$ 和 $\vec{v}$ 平行时取等 $\Rightarrow \|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$ 三角不等式

Cross product : $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta$  Area of Δ : $\frac{1}{2} \|\vec{a} \times \vec{b}\|$ 平行 : $\|\vec{a} \times \vec{b}\|$

L3-4

Plane in 3D : $ax + by + cz = d$ Normal vector : $\vec{n} = (a, b, c)$ $\vec{n} \cdot (C(x, y, z) - P_0) = 0$

Distance from point to plane : $\text{dist}(Q, \Pi) = \frac{|ax_Q + by_Q + cz_Q - d|}{\sqrt{a^2 + b^2 + c^2}}$

Parametric plane : $\vec{r}(s, t) = \vec{r}_0 + s\vec{u} + t\vec{v}$ $s, t \in \mathbb{R}$ $\vec{n} = \vec{u} \times \vec{v}$

Angle between two planes : $\cos\alpha = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|}$ $\alpha \in [0, \frac{\pi}{2}]$

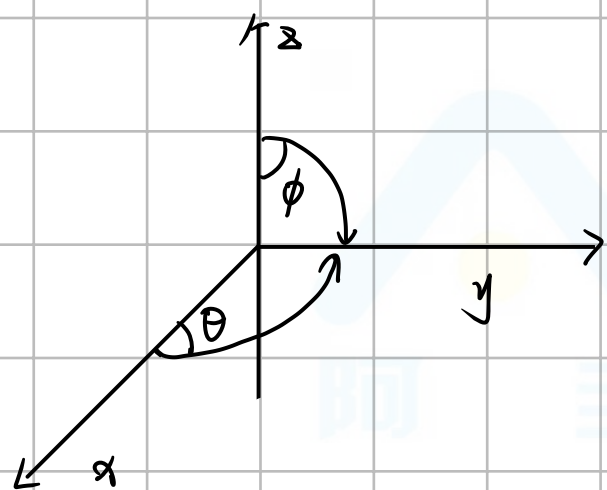
Line in 3D parametric : $\vec{r}(t) = \vec{r}_0 + t\vec{u}$
$$\begin{cases} x = x_0 + u_x t \\ y = y_0 + u_y t \\ z = z_0 + u_z t \end{cases} \Rightarrow \frac{x - x_0}{u_x} = \frac{y - y_0}{u_y} = \frac{z - z_0}{u_z}$$

交线: 两平面法向量相乘为线方向向量, 找一交点即可

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \arctan\left(\frac{y}{x}\right) \end{cases} \quad \text{If centered at } (x_0, y_0) \quad \begin{cases} x = x_0 + r \cos \theta \\ y = y_0 + r \sin \theta \end{cases} \Rightarrow \begin{cases} r = \sqrt{(x-x_0)^2 + (y-y_0)^2} \\ \theta = \arctan\left(\frac{y-y_0}{x-x_0}\right) \end{cases}$$

Cylindrical to Cartesian $\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$ Back $\begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \text{atan2}(y, x) \\ z = z \end{cases} \quad \theta \in [0, 2\pi]$

Spherical to Cartesian $\begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases}$ Back $\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \theta = \text{atan2}(y, x) \\ \phi = \arccos(z/\rho) = \text{atan2}(\sqrt{x^2 + y^2}, z) \end{cases} \quad \phi \in [0, \pi]$



$$\text{atan2}(y, x) = \begin{cases} \arctan\left(\frac{y}{x}\right), & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi, & x < 0, y \geq 0 \\ \arctan\left(\frac{y}{x}\right) - \pi, & x < 0, y < 0 \\ \frac{\pi}{2}, & x = 0, y > 0 \\ -\frac{\pi}{2}, & x = 0, y < 0 \\ \text{undefined}, & x = 0, y = 0 \end{cases}$$

Elliptic to Cartesian $\begin{cases} x = c \cosh \mu \cos \nu \\ y = c \sinh \mu \sin \nu \end{cases}$ 焦点 $(c, 0), (-c, 0)$ (μ, ν)

$$a = c \cosh \mu_0 \quad b = c \sinh \mu_0 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad c = \sqrt{a^2 - b^2}$$

Topology $\rightarrow$ 研究集合的形状性质

Open set: $U \subset \mathbb{R}^n$ 是开集, if for any $x \in U$, exist $\varepsilon > 0$ st. $B_\varepsilon(x) \subset U$

$B_\varepsilon(x) = \{y \in \mathbb{R}^n : \|y - x\| < \varepsilon\}$ called a open ball centered at x , radius with ε .

集合里面的每一个点的周围, 都能画一个小球, 而且这个小球完全待在集合里面, 即为开集. $(0, 1)$

Closed set: $F \subset \mathbb{R}^n$ 是闭集, if $\mathbb{R}^n / F$ is open $\rightarrow F \text{ closed} \Leftrightarrow F^c \text{ open}$ $[0, 1]$

Open ball: $B_r(x_0) = \{x \in \mathbb{R}^n : \|x - x_0\| < r\}$

Closed ball: $\bar{B}_r(x_0) = \{x \in \mathbb{R}^n : \|x - x_0\| \leq r\}$

Boundary: $\partial A \rightarrow$ 在其周围画任何小圆, 都会同时碰到集合里面和集合外面

$$\partial A = \bar{A} \setminus \text{int}(A) \quad \partial A = \bar{A} \cap \bar{A}^c \quad \bar{A} \rightarrow \text{Closure} \quad \text{int}(A) \rightarrow \text{Interior}$$

Interior point: If $x \in A$, exist $\varepsilon > 0$, st. $B_\varepsilon(x) \subset A$ then x called the interior point of A .

这个点周围能画一个小球, 完全待在集合里面

$$\text{Interior: } \text{int}(A) = \{x \in A : \exists \varepsilon > 0, B_\varepsilon(x) \subset A\}$$

Open set and interior: U is open $\Leftrightarrow U = \text{int}(U)$

Accumulation point: If $(B_\varepsilon(x) \setminus \{x\}) \cap A \neq \emptyset$

在 x 附近画任何小球, 都能找到别的 A 中的点 可以不属于 A
无限小的

Closure: $\bar{A} = A \cup A'$ $A' \rightarrow$ Accumulation point 的集合

$\bar{A}$ = 集合 A 加上它所有应该补上的边界点

Connect sets: $(0, 1) \checkmark$ $(0, 1) \cup (2, 3) \times$ Path-connected $\Rightarrow$ Connected

Domain: $(0, 1) \checkmark$ $(0, 1) \cup (2, 3) \times$ $[0, 1] \times$ nonempty + connect + open set

Bounded: Ω is bounded if exist $x_0 \in \mathbb{R}^n$ and $R > 0$ st. $\Omega \subset B_R(x_0)$

$(0, 1)$ 有界, 可以被 $(-1, 1)$ 包住; $\mathbb{R}$ 无界, 无法包住整个实数轴

Compact sets: K compact $\Leftrightarrow K$ closed and bounded 闭且有界