

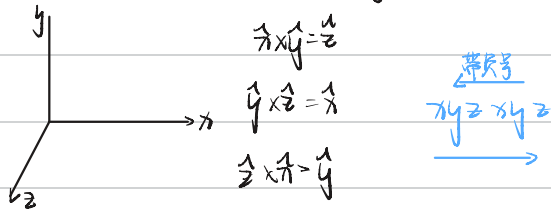
一、向量积

1) 点乘 $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \alpha = x_1 x_2 + y_1 y_2 + z_1 z_2 = \text{标量}$, $\alpha = \vec{a}$ 与 $\vec{b}$ 夹角

$$\cos \angle \vec{a} \cdot \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

2) 叉乘 $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \alpha = \text{向量}$ 方向: 右手定则, 从 $\vec{a}$ 转到 $\vec{b}$, 右手拇指指向 $\vec{c}$ 的方向

Remark: 选定 x 轴方向后, 一般用右手定则确定 y, z 轴正方向. 此时有:



二、极坐标下的圆周运动

1) $\omega = \frac{d\theta}{dt}$, $\theta(t) = \theta_0 + \int \omega dt$

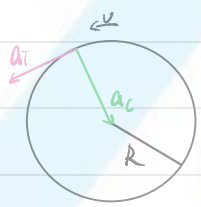
$\alpha = \frac{d\omega}{dt}$, $\omega(t) = \omega_0 + \int \alpha dt$

若 α 恒定, 则有

$\omega(t) = \omega_0 + \alpha t$

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

2) $s = R\theta \xrightarrow{\frac{ds}{dt} = \frac{d}{dt}(R\theta)} v = \text{Linear velocity} = \omega R \xrightarrow{\frac{dv}{dt} = \frac{d}{dt}(\omega R)} a_t = \text{tangential acceleration} = \alpha R$



Remark: 弧度制无单位, 角度制有

E.g. $\sin(6t)$, $6 = 6 \text{ s}^{-1} \Rightarrow 6t$ 无单位 $\Rightarrow$ 弧度制角

$\sin(16.5^\circ) \cdot t \Rightarrow$ 单位 $^\circ \Rightarrow$ 角度制角

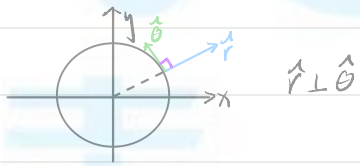
三、极坐标和单位向量

1) $\vec{r} = (\cos \theta(t) \hat{x} + \sin \theta(t) \hat{y})$

$\hat{\theta} = -\sin \theta(t) \hat{x} + \cos \theta(t) \hat{y}$

2) $\vec{v} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}$

radial velocity tangential velocity



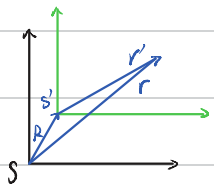
$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{\theta}$

$-r\dot{\theta}^2 \hat{r} = \text{centripetal acceleration}$

$r\ddot{\theta} \hat{\theta} = \text{tangential acceleration}$

$\ddot{r} \hat{r} = \text{radial acceleration} = \text{changes the radial velocity } \dot{r}$

四、惯性系和非惯性系



$\vec{r} = \vec{r} + \vec{r}' \xrightarrow{\text{Derivative}} \vec{v} = \vec{v} + \vec{v}' \xrightarrow{\text{Derivative}} \vec{a} = \frac{d\vec{u}}{dt} + \vec{a}'$

$\longrightarrow \vec{F}(s) = m \frac{d\vec{u}}{dt} + \vec{F}(s')$; Denote $F_{\text{fict}} = -m \frac{d\vec{u}}{dt}$

1) 简单题靠观察

2) 难题: Step 1: Draw a table Step 2: Find the easiest solution Step 3: Do the transformation

五、功、非保守力

1) 一阶导型:

$f(x) = -\alpha f(x) \longrightarrow f(x) = Ae^{-\alpha x}$

$f(x) = -\alpha f(x) + \beta \longrightarrow f(x) = Ae^{-\alpha x} + \frac{\beta}{\alpha}$

2) 二阶导型

$f(x) = -\alpha f(x) \longrightarrow f(x) = A \sin(\omega t + \varphi)$, $\omega = \sqrt{\alpha}$ ($= \sqrt{\frac{k}{m}}$) 在弹簧振子中

3) 二次型: $-b\dot{v} = ma = m \frac{dv}{dt}$, v, t 分别放两侧, 再分别积分

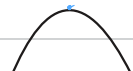
4) $W = \int_a^b \vec{F} d\vec{r} = \int_a^b (F_x dx + F_y dy + F_z dz)$

六、保守力和势能

在保守力场中, 势能函数 φ 满足 $\vec{F} = -\nabla \varphi$. 2D 中 $\varphi(x, y) = \frac{1}{2} k x^2 + \frac{1}{2} k y^2$. 3D 中 $\varphi(x, y, z) = \frac{1}{2} k x^2 + \frac{1}{2} k y^2 + \frac{1}{2} k z^2$.

1) 一维: $\vec{F} = \frac{d\varphi}{dx}$ 三维: $\vec{F} = (F_x, F_y, F_z) = (-\frac{\partial \varphi}{\partial x}, -\frac{\partial \varphi}{\partial y}, -\frac{\partial \varphi}{\partial z})$

Stable Equilibrium: $\frac{d^2 \varphi}{dx^2} \Big|_{x_0} > 0$ Unstable Equilibrium: $\frac{d^2 \varphi}{dx^2} \Big|_{x_0} < 0$



2) 简谐运动 公式必须相对平衡点列

$x = A \sin(\omega t + \varphi)$, 于弹簧振子而言: $\omega = \sqrt{\frac{k}{m_{\text{tot}}}}$, $E = \frac{1}{2} k A^2$, $A = \frac{k}{m \omega^2} x = -\omega^2 x$

*叠放振子 (叠放问题): 上物体放在下物体上, 下物体与弹簧相连, 在分离前做简谐运动

临界条件: $N = 0$; 加速度相同

*更质量弹簧振子: 在极短时间内突然放上/取下物体

特征: 在突变一瞬间, $\varphi = \int_0^t \vec{F} dt \Big|_{t=0} = 0$, 动量守恒

$m_1 v = (m_1 + m_2) u$ 放 / $(m_1 + m_2) u = m_1 u$ 取

$x = A \sin(\omega t + \varphi) \longrightarrow x = A_1 \sin(\omega_1 t + \varphi)$

A_1 : 能量守恒: $\frac{1}{2} k x_{\text{current}}^2 + \frac{1}{2} (m_1 + m_2) u^2 = \frac{1}{2} k A_1^2$

$\omega_1 = \omega = \sqrt{\frac{k}{m_{\text{tot}}}}$

φ_1 : 代入一个点

3) 保守力: 力的做功只与初末位置有关 需要证明对任意路径成立, 若路径不成立证明

七、动量定理

$\int_{t_1}^{t_2} \vec{F}_{\text{ext}} dt = \Delta \vec{p}$, if $\vec{F}_{\text{ext}} = 0 \Rightarrow \vec{p}_i = \vec{p}_f = \sum m_i \vec{v}_i$

Remark: 矢量式, 可应用在单轴上 (与质心结合)

八、质心

1) $\vec{R}_{\text{cm}} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \vec{R}_0 \cdot c + \int_0^t \vec{v}_{\text{cm}} dt$

$\vec{v}_{\text{cm}} = \frac{d\vec{R}_{\text{cm}}}{dt} = \frac{\sum m_i \vec{v}_i}{\sum m_i} = \frac{\vec{F}_{\text{tot}}}{m_{\text{tot}}} = \vec{v}_0 \cdot \text{cm} + \int_0^t \vec{a}_{\text{cm}} dt$; $K_{\text{质心}} = \frac{1}{2} m_{\text{tot}} (v_{\text{cm}})^2$

$\vec{p}_{\text{tot}} = m_{\text{tot}} \cdot \vec{v}_{\text{cm}}$

$\vec{a}_{\text{cm}} = \frac{\sum \vec{F}_{\text{ext}}}{m_{\text{tot}}}$

2) 如果动量守恒 $\longrightarrow \vec{v}_{\text{cm}} = \text{const.}$

特别地, 如果 $\vec{p}_i = \vec{p}_f = 0 \longrightarrow \vec{v}_{\text{cm}} = 0 \longrightarrow \vec{v}_{\text{cm}} = \text{const.} = \frac{\sum m_i \vec{v}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_f}{\sum m_i}$ e.g. 车上摆绳, 自车上更路

3) 如果加速度恒定, $\vec{a}_{\text{cm}} = \text{const.}$

$\vec{v}_{\text{cm}} = \vec{v}_{\text{cm},0} + \int_0^t \vec{a}_{\text{cm}} dt$ e.g. 铜球在空中爆炸, $a_{\text{cm}} = -g \hat{y}$, 若一部分速度, 若有一部分达到最高点则其 $v = 0$

4) 弹性碰撞 (m_1 碰 m_2):

$v_{1f} = \frac{(m_1 - m_2) v_1 + 2 m_2 v_2}{m_1 + m_2}$, $v_{2f} = \frac{2 m_1 v_1 + (m_2 - m_1) v_2}{m_1 + m_2}$

5) 若一个物体可以自由转动 (合力为零 + 合力矩为零), 那么物体转轴为质心