$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$-|\theta| \leq \sin\theta \leq |\theta|$ and $-|\theta| \leq 1-\cos\theta \leq |\theta|$

limit ϵ-δ definition

$\lim_{x\to a} f(x) = L$. $\forall \epsilon > 0.\ \exists \delta > 0.$ s.t if $0 < |x-a| < \delta$, then

$0 < |f(x)-L| < \epsilon$.

One-side limits formal definition.

Let $f: D \subseteq \mathbb{R} \to \mathbb{R}$

① Right-hand side

$\lim_{x\to c^+} f(x) = L$ if for every $\epsilon > 0$. there exists $\delta > 0$. st if $x \in D$

and $c < x < c+\delta$, then $|f(x)-L| < \epsilon$

② Left-hand side

$\lim_{x\to c^-} f(x) = L$ if for every $\epsilon > 0$. there exists $\delta > 0$. st if $x \in D$ and

$c-\delta < x < c$ then. $|f(x)-L| < \epsilon$

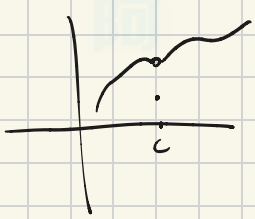
Continuity.

A function is continuous at a point $x=a$. if all three of the following conditions are satisfied:

1. $f(a)$ is defined.
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

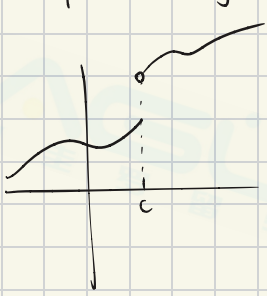
Discontinuity.

1. Removable discontinuity.



$\lim_{x\to c} f(x) \neq f(c)$.

2. Jump discontinuity.



$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$

they both exist.

3. Infinite discontinuity:

$\lim_{x\to c} f(x)$ not exists

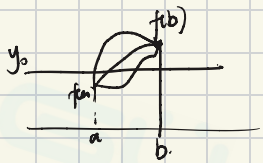
4. Oscillatory discontinuity

$\lim_{x\to c} f(x)$ not exist

Intermediate Value Theorem

If a function f is continuous on a closed interval $[a,b]$, and y_0 is any number between $f(a)$ and $f(b)$, then there exists at least one number c in (a,b). s.t

$f(c)=y_0$.



Finite limits at infinity.

$\lim_{x\to\infty} f(x)=L$ $\quad \forall \epsilon>0.\ \exists M$ st if $x>M$. then $|f(x)-L|<\epsilon$.

For example:

$\lim_{x\to\infty} \frac{1}{x}=0$ $\quad \forall \epsilon>0.\ \exists M.$ st if $x>M.$ then $|\frac{1}{x}-0|<\epsilon$

$|\frac{1}{x}|<\epsilon.$ $\quad \frac{1}{x}<\epsilon$ $\quad x>\frac{1}{\epsilon}$ $\quad$ then we choose $M=\frac{1}{\epsilon}$.

So let $\epsilon>0.$ $\quad M=\frac{1}{\epsilon}$.

when $x>M.$ $\Rightarrow x>\frac{1}{\epsilon} \Rightarrow$ $|\frac{1}{x}-0|<\epsilon$.